The University of Sydney

MATH2902 Vector Spaces

(http://www.maths.usyd.edu.au/u/UG/IM/MATH2902/)

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Tutorial 1

Let X and Y be arbitrary nonempty sets, and $f\colon X \to Y$ a function. A function $g\colon Y \to X$ is a *right inverse* of f if the composite function fg is the identity on Y. Similarly g is a *left inverse* of f if gf is the identity on X.

1. Let A be a set with 5 elements and B a set with 4 elements. Let the elements of A be called a_1, a_2, a_3, a_4 and a_5, so that $A = \{a_1, a_2, a_3, a_4, a_5\}$. Similarly let $B = \{b_1, b_2, b_3, b_4\}$.

- (i) Describe three different surjective functions with domain A and codomain B, and three different injective functions with domain B and codomain A.
- (ii) Find right inverses for each of the three surjective functions you found in (i), and left inverses for each of the injective functions.

2. Let A and B be arbitrary nonempty sets.

- (i) Let $f\colon A \to B$ be an arbitrary function. Prove that if f has a right inverse then f must necessarily be surjective, and prove that if f has a left inverse then f is necessarily injective.
- (ii) Prove that if f is surjective then it has a right inverse. Prove also that if f is injective then it has a left inverse.
- (iii) Prove that if f has both a right inverse and a left inverse then they are equal.

3. If f and g are functions with domain X and codomain Y then the correct way to prove that $f = g$ is to prove that $f(x) = g(x)$ for all $x \in X$. Similarly, if A and B are $m \times n$ matrices then proving that $A = B$ is done by proving that $A_{ij} = B_{ij}$ for all $i \in \{1, 2, \ldots, m\}$ and $j \in \{1, 2, \ldots, n\}$.

Prove that if A is an $m \times n$ matrix and I is the $n \times n$ identity matrix then $AI = A$. Prove also that if J is the $m \times m$ identity then $JA = A$.

4. Let A be an $n \times n$ matrix. A matrix B is an *inverse* of A if $AB = BA = I$. Use the previous exercise and associativity of matrix multiplication to prove that if B and C are both inverses of A then $B = C$.

5. Let F be any field. Prove that if x, $y \in F$ are such that $xy = 0$ then either $x = 0$ or $y = 0$.