The group S_4, consisting of all permutations of $\{1,2,3,4\}$, has 24 elements. It has a subgroup $K = \{(1\,2)(3\,4), (1\,3)(2\,4), (1\,4)(2\,3)\}$, and another subgroup $L = \{(1\,2), (1\,3), (2\,3), (1\,2\,3), (1\,3\,2)\}$, the former being isomorphic to $C_2 \times C_2$ and the latter to S_3. Each element of S_4 is uniquely expressible in the form lk with $l \in L$ and $k \in K$, and K is a normal subgroup, so that it follows that S_4 is isomorphic to a semidirect product of L and K. You can compute lkl^{-1} for each $l \in L$ and $k \in K$ in the following manner: simply write down the element k in cycle notation, and then permute the numbers 1, 2 and 3 according to the permutation l. Thus, for example, suppose that $l = (1\,2\,3)$ and $k = (1\,3)(2\,4)$. In the expression for k, permute 1, 2 and 3 cyclically; that is, put 2 where 1 is currently, 3 where 2 is and 1 where 3 is. This gives $(2\,1)(3\,4)$ (which is the same as $(1\,2)(3\,4)$). It can be checked that $(1\,2\,3)(1\,3)(2\,4)(1\,3\,2) = (1\,2)(3\,4)$. For example, looking at the product on the left hand side, we find that $1 \mapsto 3 \mapsto 1 \mapsto 2$, in agreement with the right hand side.

Write $\mathbb{Z}_3$ for the integers modulo 3, so that $\mathbb{Z}_3 = \{0, 1, -1\}$, where $1 + 1 = 2 = -1$. The number of 2×2 matrices over $\mathbb{Z}_3$ is $3^4 = 81$, and of these 48 have inverses. These 48 matrices form a group, known as $\mathrm{GL}(2,3)$, the *general linear group* of degree 2 over $\mathbb{Z}_3$. It has a subgroup

$$S = \{ \begin{pmatrix} a & b \\ 0 & a \end{pmatrix} \mid a \in \{\pm 1\}\, b \in \mathbb{Z}_3 \}$$

which is isomorphic to S_3, and another interesting subgroup, Q, consisting of the following eight matrices: $\pm I$, $\pm A_1$, $\pm A_2$, $\pm A_3$, where

$$A_1 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad A_2 = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \quad A_3 = \begin{pmatrix} 1 & -1 \\ -1 & -1 \end{pmatrix}.$$

These satisfy the relations $A_1^2 = A_2^2 = A_3^2 = -I$, and $A_iA_j = \pm A_k$, whenever $\begin{bmatrix} 1 & 2 & 3 \\ i & j & k \end{bmatrix}$ is a permutation of $\{1,2,3\}$, the sign being $+$ if the permutation is even, $-$ if it is odd. The group Q is known as the *quaternion group* of order 8.

The group $G = \mathrm{GL}(2,3)$ itself is another example of a semidirect product, the subgroup Q being normal, and every element of G being expressible as a product sq with $s \in S$ and $q \in Q$. Normality of Q means that $sqs^{-1} \in Q$ whenever $s \in S$ and $q \in Q$. It is possible to set up an isomorphism between S and S_3 in such a way that if $X \in S$ corresponds to the permutation σ of $\{1,2,3\}$ then for each i,

$$XA_iX^{-1} = \pm A_{\sigma i},$$

where the sign is $+$ if σ is even, $-$ if σ is odd.