MATH 402 Homework 3

Due Friday, September 21, 2018

Exercise 1.

a. *[8 points]* Suppose you have a triangle ΔABC in which $AB \neq BC$. (Without loss of generality, you can assume that $AB > BC$.) Prove that the angles opposite these sides are also not congruent, and in fact the angle opposite the longer side $\overline{AB}$ is larger than the angle opposite the shorter side $\overline{BC}$. (Hint: extend the shorter side to make an isosceles triangle. You may wish to use the Exterior Angle Theorem.)

Definition 1. Let ℓ be a line and P a point not on ℓ. Drop the perpendicular line from P to ℓ, and call the intersection point Q. We say that the *distance* from P to ℓ is the length of the segment $\overline{PQ}$.

b. *[6 points]* Prove that if Q' is any other point on ℓ, $PQ' > PQ$.

c. *[8 points]* Prove that Playfair's Postulate (or the Parallel Postulate, or other results we've already proved using them) implies the following:

*Let ℓ and m be two parallel lines, and let A, B be any two points on m. Then the distance from A to ℓ is equal to the distance from B to ℓ. (We say that ℓ and m are **equidistant**.)*

d. *[8 points]* Prove the converse: if any two parallel lines are equidistant, Playfair's Postulate must hold. (Try a proof by contradiction: suppose you have two lines m and n parallel to ℓ and both passing through some point P. Choose a point X on ℓ and think about its distance from m and n.)

Exercise 2. The point of this exercise is to reinforce the material you learned in this week's project.

a. *[2 points]* Let c be a circle with centre O and radius r. Let P be any point. Define the *power of P* with respect to the circle C.

b. *[3 points]* Prove that the power of P is positive when P is outside the circle, negative when P is inside the circle, and zero when P is on the boundary of the circle.

Exercise 3. *[10 points]* Prove the SSS similarity property: if you have two triangles such that the sides are all in proportion, the triangles are similar. (You may use the similarity properties we have already proved in lecture: AAA similarity and/or SAS similarity.)

Remember that in addition to the points assigned to each question, you will receive up to five further points for neatness and organization.