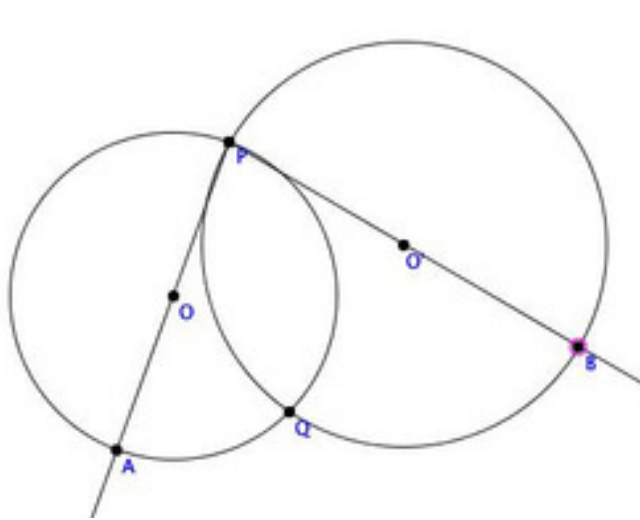


MATH 402 Homework 4

Due Friday, October 5, 2018

Exercise 1. Suppose two Euclidean circles intersect at two distinct points P and Q . (No assumption is made here about whether they intersect orthogonally or not—don't assume anything.) Draw rays from point P through the centres O and O' of the two circles, and label the points where these rays intersect the circles by A and B as below:



In this exercise you will prove that the point Q lies on the line through A and B , by completing the following steps.

- [5 points] Draw rays from the point Q through each of the five points A, O, P, O', B , and label the acute angles thus created by $\alpha, \beta, \gamma, \delta$.
- [10 points] Your drawing contains four triangles all with Q as a vertex. Determine the angles of all of these triangles, and write in the values. Justify your responses. (All of the values you find will be multiples of α, β, γ , or δ .)
- [10 points] Prove that $\alpha + \beta + \gamma + \delta = 180^\circ$, and explain why this proves that $Q \in \overleftrightarrow{AB}$.

Exercise 2. Let c be a Euclidean circle with centre O . Let A, B be two points on the boundary of c so that \overline{AB} is a chord but not a diameter.

- [10 points] Let M be the midpoint of the chord, and consider the ray from O through M . Prove that it is perpendicular to \overline{AB} .
- [10 points] Conversely, drop the perpendicular ℓ from O to \overleftrightarrow{AB} . Prove that it bisects \overline{AB} .