MATH 402 Homework 8

Due Friday 2 November, 2018

Exercise 1. Let G_1 and G_2 be two glide reflections with reflection line ℓ and m respectively.

a. *[6 points]* Suppose that ℓ and m are parallel. Show that $G_1 \circ G_2$ can be written as a translation.

b. *[7 points]* Suppose that ℓ and m are not parallel. Show that $G_1 \circ G_2$ can be written as a rotation.

Hint: Use the fact that $r_m \circ r_\ell$ is either a translation or a rotation, depending on ℓ and m. In the second case, you will also need to use the theorem from last Wednesday on composition of rotations and translations.

Exercise 2. *[10 points]* We have already seen that sometimes it is useful to take an operation f which we understand (such as rotation about $(0,0)$ or reflection across the x-axis) and use it to express more difficult operations, by performing some isometry g, applying the operation f, and then undoing the isometry g: that is, we look at the composition $f^{-1} \circ g \circ f$. This is related to an important phenomenon called *conjugation*: the *conjugate* of g by f is given by $f \circ g \circ f^{-1}$. Often the conjugate has some of the same kinds of properties as the original function g.

For example, you proved on Worksheet 5 that if you conjugate a reflection by another reflection, the result is again a reflection:

$$r_\ell \circ r_m \circ r_\ell = r_{r_\ell(m)}.$$

- Generalize this to show that if f is any isometry, $f \circ r_m \circ f^{-1} = r_{f(m)}$.

Exercise 3. Solve exercises 5.8.1–5.8.4 from Project 8. You do not need to turn in all the screenshots for this project, but you might wish to include either a sketch or a single screenshot to accompany some of the problems, to show the points, lines, and angles you use in your argument. (Don't assume the grader will just be able to guess what you mean or figure it out from the textbook!)

Here is the set-up for exercises 5.8.1–5.8.3: let A, B, E be three non-collinear points. Let m be the angle bisector of $\angle EAB$ and let n be the angle bisector of $\angle ABE$.

5.8.1: *[4 points]* Write $R_{A,\angle EAB}$ and $R_{B,\angle ABE}$ in terms of the reflections across the lines m, n, and $\overleftrightarrow{AB}$.

5.8.2: *[4 points]* Let O be the intersection of m and n. Use the results from the previous exercise to prove that $R_{B,\angle ABE} \circ R_{A,\angle EAB}$ is a rotation about O.

5.8.3: *[6 points]* By the previous exercise, we can write $R_{B,\angle ABE} \circ R_{A,\angle EAB} = R_{O,\gamma}$ for some angle γ. Prove that $\gamma = m(\angle EAB) + m(\angle ABE) \pmod{360}$. (Here's probably a good time to include a picture. Remember to take into account the orientation of the angles—if you go the wrong way around, you'll compute $180 - \gamma$ instead of γ. Remember that you have to prove this fact in general, not just check it on Geometry Explorer for the particular example you've constructed there.)

Now draw the set-up for exercise 5.8.4.

5.8.4: *[8 points]* Write the rotation at A and the rotation at B in terms of reflections across the lines $\overleftrightarrow{AL}$, $\overleftrightarrow{AB}$, $\overleftrightarrow{BN}$. Use this to prove that the composition $R_{B,\angle BAC} \circ R_{A,\angle CAB}$ is translation by twice the vector from $\overleftrightarrow{AL}$ to $\overleftrightarrow{BN}$.

Remember that in addition to the points assigned to each question, you will receive up to five further points for neatness and organization.