We say that the function is differentiable at (a, b) if the linearization is a "good enough" approximation of f near (a, b). The difference between the linearization L and the function f at a point $(x + \Delta x, y + \Delta y)$ is the error, $E(\Delta x, \Delta y)$.

What are some ways of checking if a function is differentiable?

I. Check if the error goes to zero: $\lim_{(\Delta x, \Delta y)\to(0,0)} E(\Delta x, \Delta y) = 0$?

II. Check if the error is small relative to the change $(\Delta x, \Delta y)$:

$$\lim_{(\Delta x, \Delta y)\to(0,0)} \frac{E(\Delta x, \Delta y)}{\sqrt{(\Delta x)^2 + \Delta y)^2)}} = 0?$$

III. Check if the differentials f_x, f_y exist at (a, b).

IV. Check if the differentials f_x, f_y exist on a disk containing (a, b) and are continuous at (a, b).

V. Zoom in on the graph of the function. See if the graph of the function starts to look like the tangent plane (i.e. the graph of the linearization).

Choose an option:

(a) Any of these will work.

(b) Only I or II will work.

(c) Anything except V will work.

(d) II, and V will work; IV can be used to prove that a function *is* differentiable, but it can't be used to prove that a function *isn't* differentiable.

Practice with the chain rule

Let $W(t) = F(u(t), v(t))$. Assume that we know the following:

- $u(1) = 0, v(1) = 3$
- $u'(1) = 2, v'(1) = 0$
- $F_u(0,3) = 1, F_u(2,0) = -1, F_v(2,0) = 1$

Find $\frac{dW}{dt}(1)$.

(a) -2

(b) 2

(c) We don't have enough information.

(d) I don't know how to do this.

Practice with the chain rule

Suppose $x(s,t)$ and $y(s,t)$ are differentiable functions of two variables. Consider

$$\begin{aligned} f : \mathbb{R}^2 &\to \mathbb{R} \\ (x,y) &\mapsto \cos(x+y). \end{aligned}$$

Let $h(t) = f(x(s,t), y(s,t))$. Suppose that (s_0, t_0) is a point of $\mathbb{R}^2$ such that $x(s_0,t_0) = y(s_0,t_0) = \pi$. What can you say about $\frac{\partial h}{\partial x}(s_0,t_0)$?

(a) It depends on x and y.

(b) It's 1.

(c) It's 0.

(d) I don't understand the question.