Last time: linear change of coordinates

Recall that for a linear transformation $T : \mathbf{R}^2 \to \mathbf{R}^2$ with Jacobian

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} x_u & x_v \\ y_u & u_v \end{vmatrix}$$

we have the following formula:

$$\iint_{T(D)} f(x,y)dA = \iint_D f(T(u,v)) \left| \frac{\partial(x,y)}{\partial(u,v)} \right| dA$$

Use this to calculate the area of the ellipse $B = \{\frac{x^2}{\alpha^2} + \frac{y^2}{\beta^2} \leq 1\}$, by finding a linear transformation T with $T(D) = B$, where B is the unit disk $\{u^2 + v^2 \leq 1\}$.

- a I don't know what to do.
- b I found T, but now I don't know what to do.
- c I found T and the Jacobian, but I'm stuck now.
- d I'm done.

Practice with image and one-to-one

Let $T(r,\theta) = (r\cos\theta, r\sin\theta)$.
Let $D = [0,\infty) \times [0,2\pi)$. Is the image of T all of $\mathbb{R}^2$? Is T one-to-one on D?

(a) Yes and yes.

(b) Yes and no.

(c) No and yes.

(d) No and no.

(e) I don't know.

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(b) Yes and no.

(c) No and yes.

(d) No and no.

(e) I don't know.

Practice with the Jacobian

Let $T(u, v) = (\frac{u^2}{v}, \frac{v}{u})$. Find $\frac{\partial(x,y)}{\partial(u,v)}$.

(a) $\frac{3}{v}$
(b) $\frac{1}{v}$
(c) $2v + \frac{1}{v}$
(d) $u + v$
(e) I don't know how.

Change of coordinates in three-dimensions

Theorem

Let T be a transformation from $D \subset \mathbb{R}^3$ to $\mathbb{R}^3$ such that

- *D and $T(D)$ are "nice";*
- *$\frac{\partial(x,y,z)}{\partial(u,v,w)} \neq 0$ and T is one-to-one on D except possibly on the boundary.*

Suppose f is a continous function on $T(D)$. Then

$$\iiint_{T(D)} f(x,y,z)dV_{xyz} = \iiint_D f(T(u,v,w)) \left| \frac{\partial(x,y,z)}{\partial(u,v,w)} \right| dV_{uvw}.$$