MATH2065: INTRO TO PDEs

Fourier Transforms Table

Function	Fourier Transform
$f(x) = \mathcal{F}^{-1}\{F(\omega)\} = \int_{-\infty}^{\infty} F(\omega) e^{-i\omega x}\, \mathrm{d}\omega$	$F(\omega) = \mathcal{F}\{f(x)\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(x)\, e^{i\omega x}\, \mathrm{d}x$
$e^{-\alpha x^2} \quad (\alpha > 0)$	$\frac{1}{\sqrt{4\pi\alpha}} e^{-\omega^2/4\alpha}$
$\sqrt{\frac{\pi}{\beta}} e^{-x^2/4\beta} \quad (\beta > 0)$	$e^{-\beta\omega^2}$
$\frac{2\alpha}{x^2+\alpha^2} \quad (\alpha > 0)$	$e^{-\alpha\|\omega\|}$
$e^{-\alpha\|x\|} \quad (\alpha > 0)$	$\frac{1}{\pi}\frac{\alpha}{\omega^2+\alpha^2}$
$f(x) = \begin{cases} 0 & \|x\| > \alpha \\ 1 & \|x\| < \alpha \end{cases}$	$\frac{1}{\pi}\frac{\sin \alpha\omega}{\omega}$
$\delta(x - x_0)$ (Dirac delta)	$\frac{1}{2\pi} e^{i\omega x_0}$
$a\, f(x) + b\, g(x)$	$a\, F(\omega) + b\, G(\omega)$ (linearity)
$f(x - \beta)$	$e^{i\omega\beta} F(\omega)$ (x-shifting)
$f(x)\, e^{-i\beta x}$	$F(\omega - \beta)$ (ω − shifting)
$x^n\, f(x)$	$(-i)^n \frac{d^n}{d\omega^n} F(\omega)$ (ω − derivatives)
$\frac{d^n f}{dx^n}$	$(-i\omega)^n\, F(\omega)$ (x-derivatives)
$f \star g = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(x-\bar{x})\, g(\bar{x})\, \mathrm{d}\bar{x}$	$F(\omega)\, G(\omega)$ (convolution)